Math 202(01) Solution
Quiz 6 5 April 2002

Show all work. An answer without sufficient work shown may not receive full credit even if it is correct.

1. Set up, but do not evaluate, a definite integral for the volume generated when the region bounded by the curve $y = \sqrt{x}$ and the line $y = x/2$ is revolved about the y-axis. Use the washer method.

   Solution: The curve and the line intersect at $(0, 0)$ and $(4, 2)$. We stack the washers along the y-axis from $y = 0$ to $y = 2$. The outer radius of the washer at height y is determined by the line $y = x/2$, or $x = 2y$. The inner radius of the washer at height y is determined by the curve $y = \sqrt{x}$ or $x = y^2$. We get

$$\begin{aligned} V &= \int_0^2 \pi((2y)^2 - (y^2)^2)\, dy \\ &= \int_0^2 \pi(4y^2 - y^4)\, dy. \end{aligned}$$

2. Let R be the region in the xy plane that lies to the right of the y-axis and is bounded by the curve $y = x^2$ and the lines $y = -x$ and $x = 2$. Set up, but do not evaluate, a definite integral for the volume generated when R is revolved about the line $x = 3$. Use the shells method.

   Solution: The axis of rotation is $x = 3$, and since the shells are centered on the line $x = 3$, we will use dx for the thickness of a shell. The radius of a shell at position x is $3 - x$, and the height is determined by the curves $y = x^2$ (on top) and $y = -x$ (on the bottom). We get

$$\begin{aligned} V &= 2\pi \int_0^2 (3 - x)(x^2 - (-x))\, dx \\ &= 2\pi \int_0^2 (3 - x)(x^2 + x)\, dx. \end{aligned}$$

3. Set up, but do not evaluate, a definite integral for the area of the surface generated when the part of the curve $y = \dfrac{1}{\ln x}$ between $x = 2$ and $x = 10$ is revolved about the x-axis.

Solution: We have $y = \dfrac{1}{\ln x}$ so that $y' = -\dfrac{1}{x(\ln x)^2}$. Plugging this into the formula for the area of a surface of revolution, we get

$$A = 2\pi \int_2^{10} \frac{1}{\ln x} \sqrt{1 + \frac{1}{x^2(\ln x)^4}}\, dx.$$