Math 202(01) — Solution

Quiz 7 — 12 April 2002

Show all work. An answer without sufficient work shown may not receive full credit even if it is correct.

1. Find a formula for the general term a_n of the sequence $\left\{1, \frac{3}{4}, \frac{5}{9}, \frac{7}{16}, \frac{9}{25}, \ldots\right\}$, assuming that the pattern of the first few terms continues.

   Solution: The numerators appear to follow the pattern $1, 3, 5, 7, 9, \ldots$, an arithmetic progression with a common difference of 2. To make the numerator of $a_1 = 1$, we take $2n - 1$ as our formula for the numerator.

   The denominators appear to be the squares of the integers: $1^2, 2^2, 3^2, 4^2, \ldots$. We take n^2 as the formula for the denominator.

   We propose $a_n = \dfrac{2n-1}{n^2}$ as the formula for the general term of the sequence.

2. Consider the sequence $\{a_n\}_{n=1}^{\infty}$ where

$$a_n = \frac{n^2 - 2n}{3n^2 + 5}.$$

   Find $\lim_{n\to\infty} a_n$.

   Solution: We multiply top and bottom by $\dfrac{1}{n^2}$ to get

$$\begin{aligned} \lim_{n\to\infty} \frac{n^2 - 2n}{3n^2 + 5} &= \lim_{n\to\infty} \frac{1 - \frac{2}{n}}{3 + \frac{5}{n^2}} \\ &= \frac{1}{3}. \end{aligned}$$

3. Consider the series $\sum_{n=0}^{\infty} \dfrac{31}{10^n}$. Write the first five partial sums of this series (that is s_0, s_1, s_2, s_3, and s_4) as decimals.

   Solution: We have

$$\begin{aligned} s_0 &= 31 &&= 31 \\ s_1 &= 31 + 3.1 &&= 34.1 \\ s_2 &= 34.1 + 0.31 &&= 34.41 \\ s_3 &= 34.41 + 0.031 &&= 34.441 \\ s_4 &= 34.441 + 0.0031 &&= 34.4441. \end{aligned}$$