Math 202(01) Solution
Quiz 8 19 April 2002

Show all work. An answer without sufficient work shown may not receive full credit even if it is correct.

1. Determine the sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^{n-1}}{3^{n+1}}$.

   Solution: The first few terms of the series are

$$\frac{1}{9} - \frac{2}{27} + \frac{4}{81} - \frac{8}{243} + \cdots$$

   so we have a series of the form $\displaystyle\sum_{n=0}^{\infty} ar^n$ with $a = \frac{1}{9}$ and $r = -\frac{2}{3}$. The sum of the series is

$$\frac{a}{1-r} = \frac{\frac{1}{9}}{1+\frac{2}{3}} = \frac{1}{15}.$$

2. Determine whether the series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ is convergent or divergent.

   Solution: We apply the integral test. The terms $\dfrac{1}{n(\ln n)^2}$ form a sequence that is clearly decreasing and has limit 0, so we need only compute

$$\begin{aligned}\int_2^{\infty} \frac{dx}{x(\ln x)^2} &= \lim_{t\to\infty} \left[-\frac{1}{\ln x}\right]_2^t \\ &= \lim_{t\to\infty} \left[\frac{1}{\ln 2} - \frac{1}{\ln t}\right] \\ &= \frac{1}{\ln 2}.\end{aligned}$$

   The integral is convergent, and so the series converges, as well.

3. Determine whether the series $\displaystyle\sum_{n=0}^{\infty} \frac{3^n}{n+4^n}$ converges or diverges.

   Solution: We use the Basic Comparison Theorem. For $n \geq 0$, we know that

$$\frac{3^n}{n+4^n} \leq \frac{3^n}{4^n}$$

   We also know that $\displaystyle\sum_{n=0}^{\infty} \frac{3^n}{4^n}$ converges (it's a geometric series with $|r| < 1$), so by BCT, the given series converges as well.