1. Determine the sum of the series \( \sum_{n=1}^{\infty} \frac{(-2)^{n-1}}{3^{n+1}}. \)

Solution: The first few terms of the series are 
\[
\frac{1}{9} - \frac{2}{27} + \frac{4}{81} - \frac{8}{243} + \cdots
\]
so we have a series of the form \( \sum_{n=0}^{\infty} ar^n \) with \( a = \frac{1}{9} \) and \( r = -\frac{2}{3} \). The sum of the series is 
\[
\frac{a}{1-r} = \frac{\frac{1}{9}}{1 + \frac{2}{3}} = \frac{1}{15}.
\]

2. Determine whether the series \( \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2} \) is convergent or divergent.

Solution: We apply the integral test. The terms \( \frac{1}{n(\ln n)^2} \) form a sequence that is clearly decreasing and has limit 0, so we need only compute
\[
\int_2^{\infty} \frac{dx}{x(\ln x)^2} = \lim_{t \to \infty} \left[ -\frac{1}{\ln x} \right]_2^t = \lim_{t \to \infty} \left[ \frac{1}{\ln 2} - \frac{1}{\ln t} \right] = \frac{1}{\ln 2}.
\]
The integral is convergent, and so the series converges, as well.

3. Determine whether the series \( \sum_{n=0}^{\infty} \frac{3^n}{n+4^n} \) converges or diverges.

Solution: We use the Basic Comparison Theorem. For \( n \geq 0 \), we know that 
\[
\frac{3^n}{n+4^n} \leq \frac{3^n}{4^n}
\]
We also know that \( \sum_{n=0}^{\infty} \frac{3^n}{4^n} \) converges (it’s a geometric series with \( |r| < 1 \)), so by BCT, the given series converges as well.