

Here are some interesting problems you can solve using the techniques in §§ 6.2, 6.3, and 9.2.

1. Let r and h be positive constants. If the triangle bounded by the x -axis, the y -axis, and the lines $y = \frac{r}{h}x$ and $x = h$ is revolved about the x -axis, the resulting solid is a (right circular) cone with radius r and height h .

Use an integral (with either disks or shells) to derive the formula for the volume of a cone with radius r and height h .

2. Find a formula for the surface area of a cone with radius r and height h .
3. A sphere of radius R can be viewed as a solid of revolution: take the area under the curve $y = \sqrt{R^2 - x^2}$ between $x = -R$ and $x = R$, and revolve it about the x -axis.

You can now use an integral to derive the formula for the volume of a sphere of radius R . Stewart does this using the disk method in Example 1 of § 6.2. Try to get the same result using the shell method.

4. Find a formula for the surface area of a sphere of radius R .
5. A *bead* is a sphere with a cylindrical hole drilled through its center. A bead is determined by two dimensions: the radius of the sphere (which we will denote R) and the radius of the hole (which we will denote r). To construct a bead as a solid of revolution, take the region bounded by the curve $y = \sqrt{R^2 - x^2}$, the curve $y = -\sqrt{R^2 - x^2}$, and the line $x = r$, and revolve it about the y -axis.

Use the shell method to find a formula for the volume of a bead with sphere radius R and hole radius r .

(Your answer should be a constant multiple of $(R^2 - r^2)^{\frac{3}{2}}$. Where does this dimension appear in the bead? See § 6.3, Problem 44.)

6. § 6.2, Problem 59: find a formula for the volume of a torus.