

Reading: NZM §2.1.

Exercises: Write your solutions in complete sentences.

1. (§2.1, problem 15) Find integers a_1, a_2, a_3, a_4, a_5 such that every integer x satisfies at least one of the congruences $x \equiv a_1 \pmod{2}$, $x \equiv a_2 \pmod{3}$, $x \equiv a_3 \pmod{4}$, $x \equiv a_4 \pmod{6}$, $x \equiv a_5 \pmod{12}$. Explain how you know your answer works, and, if applicable, how you found it.
2. (a) Let p and q be primes. Prove that if $p \equiv 1 \pmod{q-1}$, then $a^p \equiv a \pmod{q}$ for every integer a .
(b) (§2.1, problem 20) Use the result in part (2a) to prove that $n^7 - n$ is divisible by 42 for any integer n .
3. (§2.1, problem 27) Prove that $\frac{n^5}{5} + \frac{n^3}{3} + \frac{7n}{15}$ is an integer for every integer n .
4. (§2.1, problem 46) Show that for any prime p , if $a^p \equiv b^p \pmod{p}$, then $a^p \equiv b^p \pmod{p^2}$.
5. (§2.1, problem 50) For every positive integer n , prove that there exists a (non-zero) multiple m of n whose base-ten representation contains only the digits 0 and 1. Prove that the same holds for the digits 0 and 2, for 0 and 3, and so on up to the digits 0 and 9, but for no other pair of digits.

Cultural aside:

My name is Eugene Debs Hartke, and I was born in 1940. I was named at the behest of my grandfather, ... in honor of Eugene Debs of Terre Haute, Indiana. Debs was a Socialist and a Pacifist and a Labor Organizer who ran several times for the Presidency of the United States of America, and got more votes than has any other candidate nominated by a third party in the history of this country.

Debs died in 1926, when I was negative 14 years of age.

The year is 2001 now.

If all had gone the way a lot of people thought it would, Jesus Christ would have been among us again, and the American flag would have been planted on Venus and Mars.

No such luck!

At least the world will end, an event anticipated with great joy by many. It will end very soon, but not in the year 2000, which has come and gone. From that I conclude that God Almighty is not heavily into Numerology.

Kurt Vonnegut, *Hocus Pocus*