Math 319(01) Solution
Quiz 6 5 April 2002

1. State the Chinese Remainder Theorem.

   Answer: Let $m_1, m_2, \ldots, m_r$ be positive integers, relatively prime in pairs, and let $a_1, a_2, \ldots, a_r$ be any integers. Then there is a solution to the system

$$\begin{aligned} x &\equiv a_1 \pmod{m_1} \\ x &\equiv a_2 \pmod{m_2} \\ &\vdots \\ x &\equiv a_r \pmod{m_r}. \end{aligned}$$

   Furthermore, if x_0 is one such solution, then x is a solution if and only if $x = x_0 + km$ for some k, where $m = m_1 m_2 \cdots m_r$.

2. Determine $\varphi(99,000,000)$. You may want to know that

$$99,000,000 = 2^6 \times 3^2 \times 5^6 \times 11.$$

   (Brownie points if you can do this without a calculator.)

   Solution: We have

$$\begin{aligned} \varphi(99,000,000) &= \varphi(2^6)\varphi(3^2)\varphi(5^6)\varphi(11) \\ &= (2^6 - 2^5) \times (3^2 - 3) \times (5^6 - 5^5) \times (10) \\ &= 2^5 \times (2-1) \times 3 \times (3-1) \times 5^5 \times (5-1) \times 10 \\ &= 10^6 \times 3 \times 2 \times 4 \\ &= 24 \times 10^6 \\ &= 24,000,000. \end{aligned}$$