1. State the Chinese Remainder Theorem.

Answer: Let $m_1, m_2, \ldots, m_r$ be positive integers, relatively prime in pairs, and let $a_1, a_2, \ldots, a_r$ be any integers. Then there is a solution to the system

$$
x \equiv a_1 \pmod{m_1}
$$

$$
x \equiv a_2 \pmod{m_2}
$$

$$
\vdots
$$

$$
x \equiv a_r \pmod{m_r}.
$$

Furthermore, if $x_0$ is one such solution, then $x$ is a solution if and only if $x = x_0 + km$ for some $k$, where $m = m_1 m_2 \cdots m_r$.

2. Determine $\varphi(99,000,000)$. You may want to know that

$$
99,000,000 = 2^6 \times 3^2 \times 5^6 \times 11.
$$

(Brownie points if you can do this without a calculator.)

Solution: We have

$$
\varphi(99,000,000) = \varphi(2^6)\varphi(3^2)\varphi(5^6)\varphi(11)
$$

$$
= (2^6 - 2^5) \times (3^2 - 3) \times (5^6 - 5^5) \times (10)
$$

$$
= 2^5 \times (2 - 1) \times 3 \times (3 - 1) \times 5^5 \times (5 - 1) \times 10
$$

$$
= 10^6 \times 3 \times 2 \times 4
$$

$$
= 24 \times 10^6
$$

$$
= 24,000,000.
$$