Math 139 Problem Set #2 Due 18 February 2003

Reading: Barr, §§2.1, 2.2.

Exercises: Write your solutions clearly, remembering that they will be graded for presentation as well as correctness. Please prepare *separate* solution sets for the A problems and the B problems. You will hand them in in different places.

A1. Calculate the following:

(a) $55 \bmod 13$

(b) $81 \bmod 13$

(c) $(55 \times 81) \bmod 13$

(d) $-4 \bmod 15$

A2. (a) Find three numbers x satisfying $x \equiv 19 \pmod{32}$

(b) Find three numbers x satisfying both $x \equiv 0 \pmod{2}$ and $x \equiv 1 \pmod{13}$.

(c) Find three numbers x satisfying both $x \equiv 1 \pmod{3}$ and $x \equiv 1 \pmod{10}$.

A3. (a) Write down a complete multiplication table modulo 8. Index the rows and columns with the numbers 0 through 7, and in the cell at row x and column y, fill in $(x \times y) \bmod 8$.

(b) Solve the congruence $3x \equiv 7 \pmod{8}$. Find a solution in the set $\{0, 1, 2, \ldots, 8\}$.

(c) (Trickier) Are there any solutions (again in the set $\{0, 1, 2, \ldots, 7\}$) to the congruence $6x \equiv 20 \pmod{8}$? How about $6x \equiv 21 \pmod{8}$?

B1. (a) Which of the numbers in the set $\{0, 1, 2, \ldots, 21\}$ have multiplicative inverses modulo 22?

(b) Find the multiplicative inverse of each number you identified in part B1a.

(c) Solve the congruences

i. $17x + 14 \equiv 2 \pmod{22}$

ii. $6x - 4 \equiv x + 10 \pmod{22}$

(d) Given that

$$\begin{aligned} 3a &+ b \equiv 15 \pmod{22} \\ 8a &+ b \equiv 6 \pmod{22} \end{aligned}$$

find a and b.

B2. Decrypt the following, using the affine decryption key $a = 5$, $b = 7$.

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JMCRZ CJXSV WJSRC XDRCL VXWPI PCURW P
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