

1. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ by $f(x) = 7x - 4$. Find a function g so that $g \circ f(x) = x$ for all x .

Solution: The function g must add 4 to its input and then divide by 7. We get

$$g(x) = \frac{x + 4}{7}.$$

2. Find multiplicative inverses for 2 and 3 modulo 7.

Solution: By trial and error, we find that $2 \times 4 \equiv 1 \pmod{7}$ and $3 \times 5 \equiv 1 \pmod{7}$.

The multiplicative inverse of 2 mod 7 is 4;
the multiplicative inverse of 3 mod 7 is 5.

3. Solve the congruence $9x + 15 \equiv 10 \pmod{89}$. Find a solution in the set $\{0, 1, 2, \dots, 88\}$.
(HINT: $9 \times 10 = 90$.)

Solution: We subtract 15 from both sides to get

$$\begin{aligned} 9x &\equiv -5 \pmod{89} \\ &\equiv 84 \pmod{89}. \end{aligned}$$

By the hint, we know that the multiplicative inverse of 9 is 10, so we multiply both sides by 10 to get

$$x \equiv 840 \pmod{89}$$

Finally, we find that $840 \bmod 89 = 39$.

The solution we're looking for is $x = 39$.