

1. Let $\pi(n)$ denote the prime counting function. Given that $\pi(100) = 25$, find $\pi(105)$.

Solution: We need only determine how many primes there are in the interval $100 < n \leq 105$. Since 102, 104, and 105 are obviously composite, we need to check only 101 and 103. By trial division, we find that both are prime, so $\pi(105) = 27$.

2. According to the Prime Number Theorem, about how many seven-digit primes are there?

The number of seven-digit primes is $\pi(9,999,999) - \pi(999,999)$, which, by the Prime Number Theorem, is about

$$\frac{9,999,999}{\ln(9,999,999)} - \frac{999,999}{\ln(999,999)}$$

The calculator says this is about 548,038.