Math 139 | Solution
Quiz 7 | 11 April 2003

1. Find integers s and t such that $232s + 45t = 1$.

   Solution: First we carry out the steps in the Euclidean algorithm. We get

$$\begin{aligned} 232 &= 5 \times 45 + 7 \\ 45 &= 6 \times \ \ 7 + 3 \\ 7 &= 2 \times \ \ 3 + 1 \end{aligned}$$

   Now we begin the back-substitution. We get

$$\begin{aligned} 1 &= 7 - 2 \times 3 \\ &= 7 - 2(45 - 6 \times 7) \\ &= -2 \times 45 + 13 \times 7 \\ &= -2 \times 45 + 13(232 - 5 \times 45) \\ &= 13 \times 232 - 67 \times 45. \end{aligned}$$

   That's the answer: $13 \times 232 - 67 \times 45 = 1$.

2. Compute $17^{41} \bmod 100$.

   Solution: We begin by computing powers of 17 modulo 100. We have

$$\begin{aligned} 17^1 &\equiv 17 \pmod{100} \\ 17^2 &\equiv 89 \pmod{100} \\ 17^4 &\equiv 21 \pmod{100} \\ 17^8 &\equiv 41 \pmod{100} \\ 17^{16} &\equiv 81 \pmod{100} \\ 17^{32} &\equiv 61 \pmod{100} \end{aligned}$$

   Next we observe that $41 = 32 + 8 + 1$, so that

$$17^{41} \equiv 17^{32} \times 17^8 \times 17^1 \equiv 61 \times 41 \times 17 \pmod{100}$$

   We get $42517 \bmod 100 = 17$.