

**Reading:** B & D, Chapter 1.

**Exercises:** Write your solutions clearly, and in complete sentences, remembering that they will be graded for presentation as well as correctness. Please prepare **separate** solution sets to the *A* problems, the *B* problems, and the *C* problems. You will hand them in in different places.

**A1.** For each operator, either prove that the operator is linear or find an example showing that it is not linear.

(a)  $F_1[y] = 2y'' + y$ .

(b)  $F_2[y] = e^y$ .

(c)  $F_3[y] = y^2 + y'$ .

**A2.** Verify that  $y = te^{-2t} + t - 1$  is a solution to  $y'' + 4y' + 4y = 4t$ .

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**B1.** For what values of  $\alpha$  is  $y(t) = \cos(\alpha t)$  a solution to  $y'' + 5y = 0$ ?

**B2.** For what values of  $r$  is  $y(t) = e^{rt}$  a solution to  $y'' + 3y' - 10y = 0$ ?

**B3.** For what values of  $r$  is  $y(t) = t^r$  a solution to  $t^2y'' + 3ty' - 8y = 0$ ?

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**C1.** By hand, draw direction fields for the following first-order differential equations.

(a)  $\frac{dy}{dt} = y - \frac{t}{2}$

(b)  $\frac{dy}{dt} = \frac{(t-2)^2}{2} - y$

Use as grid points the integer lattice points in the  $ty$ -plane with  $0 \leq t \leq 4$  and  $0 \leq y \leq 4$ .

Sketch a few solution curves on each direction field. Try to predict the long-term behavior of solutions to these differential equations.

**C2.** Use Maple to draw direction fields for the following first-order differential equations.

(a)  $\frac{dy}{dt} = y^3 - 2y$

(b)  $\frac{dy}{dt} = \cos(\pi t) - y$

Hand in printouts of each Maple plot. Sketch a few solution curves on each one. Use the region  $0 \leq t \leq 6$  and  $-2 \leq y \leq 2$  in the  $ty$ -plane. Try to predict the long-term behavior of solutions to these differential equations.

Here's what you need to know about Maple. Start by loading the differential equations package with the command

```
> with(DEtools):
```

To draw a direction field, use the command `dfieldplot`. Here's the syntax for part C2a:

```
> dfieldplot(diff(y(t),t)=y(t)^3-2*y(t),y(t),t=0..6,y=-2..2);
```