Math 333 Problem Set #5 Due 14 March 2003, 12:00 NOON

Reading: B & D, §§ 3.1, 3.4

Exercises: Write your solutions clearly, and in complete sentences, remembering that they will be graded for presentation as well as correctness.

A1. The solutions to the following four second-order initial value problems are all so common that they have their own names. For each problem below, find the solution in terms of exponential functions (possibly with imaginary exponents), and then give the solution's more common name. (You'll recognize them – for example, the common name of the solution to the first problem is cosine.)

(a) $y'' + y = 0; \quad y(0) = 1, \quad y'(0) = 0.$

(b) $y'' + y = 0; \quad y(0) = 0, \quad y'(0) = 1.$

(c) $y'' - y = 0; \quad y(0) = 1, \quad y'(0) = 0.$

(d) $y'' - y = 0; \quad y(0) = 0, \quad y'(0) = 1.$

B1. Consider the initial value problem

$$y'' + 3y' - 4y = 0; \quad y(0) = 1, \quad y'(0) = \alpha.$$

(a) Solve the initial value problem. Your solution will depend on the parameter α.

(b) The behavior of the solution as $t \to \infty$ also depends on α.

i. For what values of α does the solution to the initial value problem go to $+\infty$ as $t \to \infty$? (Your answer should be an interval in the real line.)

ii. For what values of α does the solution go to $-\infty$?

iii. For what values of α does the solution remain bounded? What is the limiting value?

B2. (a) Let α denote a positive constant. Solve the initial value problem

$$2y'' + 5y' - 3y = 0; \quad y(0) = 3, \ y'(0) = -\alpha.$$

(b) Now let $\alpha = 2$ in your solution to part B2a, and show that the solution has a global minimum. Find the value of t where the global minimum occurs. (Give the exact value.)

(c) What is the smallest value of α for which your solution in part (B2a) has no local minimum?

C1. (a) Assume that $|b| < 2$. Find the (real-valued) general solution to the differential equation

$$y'' + by' + y = 0.$$

You should get a family of solutions including two constants c_1 and c_2, and depending on the parameter b.

(b) Describe in qualitative terms (increasing, decreasing, oscillating, and so on) the general behavior of your family of solutions for

i. $b = 1$.

ii. $b = 0$.

iii. $b = -1$.

Plot a typical solution for each of the given values of b. (Choose any convenient values for c_1 and c_2 (except $c_1 = c_2 = 0$).)

(c) What happens to your solutions as b increases toward 2? What happens as b decreases toward -2?