Math 333 Problem Set #6 Due 1 April 2003, 5:00 PM

Reading: B & D, §§ 3.3, 3.4, 3.5, 3.6

Exercises: Write your solutions clearly, and in complete sentences, remembering that they will be graded for presentation as well as correctness.

A1. Solve the initial value problem $y'' + 4y' + 4y = 0; \quad y(0) = 1,\ y'(0) = 0.$

A2. Consider the third-order differential equation

$$y''' + 6y'' + 12y' + 8y = 0. \tag{1}$$

The characteristic equation of (1) is $(r + 2)^3 = 0$, and, as you can check, $y_1 = e^{-2t}$ is a solution.

(a) Find two other solutions, y_2 and y_3, that are not scalar multiples of y_1 or of each other.

(Probably the best way to do this is to make guesses and then plug them in to (1) to see if they work.)

(b) Solve the third-order initial value problem

$$y''' + 6y'' + 12y' + 8y = 0; \qquad y(0) = 1, \quad y'(0) = -5, \quad y''(0) = 20.$$

B1. (B & D, §3.5, problem 24) Given that $y_1 = t$ is one solution to the differential equation $t^2y'' + 2ty' - 2y = 0$, (with $t > 0$), use the method of reduction of order to find a second solution, y_2.

Then calculate $W(y_1, y_2)$ to be sure that y_2 is truly independent of y_1.

B2. Carry out the calculation of $W(y_1, y_2)$ with $y_1 = e^{\lambda t}\cos(\mu t)$ and $y_2 = e^{\lambda t}\sin(\mu t)$.

C1. Solve the initial value problem $y'' + y' - 2y = t^2 + e^{3t}; \quad y(0) = 1,\ y'(0) = 2.$

C2. Recall that if $y(t)$ gives the position of an object at time t, then $y''(t)$ is the object's acceleration. Also recall Newton's law: the acceleration of an object is a constant (namely, $1/m$) times the force acting on the object. The initial value problem

$$y'' = \cos t - \beta y'; \quad y(0) = 0,\ y'(0) = 0 \tag{2}$$

thus models an object (with mass 1) moving under the influence of an outside force $\cos t$ and a second force $-\beta y'$ that pushes against the object's motion, with a magnitude proportional to the object's velocity (we assume $\beta > 0$). The second force is a reasonable way to model sliding friction. The parameter β is the coefficient of friction.

(a) Solve the initial value problem (2). All the constants in your solution will depend on β

(b) Use the computer to plot solutions to (2) with $\beta = 0.25$, $\beta = 4$, and $\beta = 8$. With each solution plot, also plot the function $\cos t$ (the forcing function).

(c) Comment on the relation between the forcing function and the response function (that is, the solution) for the various values of β. How do the amplitude, frequency, and phase shift of the response seem to depend on β?