Math 333 Problem Set #7 Due 18 April 2003, 12:00 NOON

Reading: B & D, §§3.6 – 3.9

Exercises: Write your solutions clearly, and in complete sentences, remembering that they will be graded for presentation as well as correctness.

A1. (a) Solve the initial value problem

$$y'' + 2y' + 10y = 37\cos 3t; \quad y(0) = 0, \quad y'(0) = 1.$$

Identify which terms in your solution belong to the transient solution and which belong to the steady-state solution.

(b) Compare the amplitude and phase angle of the steady-state solution with those of the function $37\cos 3t$.

A2. Consider a mass and spring system with no damping, driven by a sinusoid forcing function at 500 cycles per second (nb. *cycles*, not radians). In the response, we observe beats. The amplitude of the response function rises and falls two times per second.

Suppose $m = 100$ grams. Find the possible values for k, the spring constant.

B1. (B & D, §3.9, problem 17) Consider a vibrating system described by the initial value problem

$$u'' + \frac{1}{4}u' + 2u = 2\cos\omega t, \quad u(0) = 0, \quad u'(0) = 2.$$

(a) Determine the steady-state part of the solution of this problem.

(b) Find the amplitude A of the steady-state solution in terms of ω.

(c) Plot A as a function of ω.

(d) Find the maximum value of A and the frequency ω for which it occurs.

B2. Consider a series circuit with a 10 microfarad capacitor (a microfarad is 10^{-6} farad), a 0.25 henry inductor, and a 10 ohm resistor.

(a) Assume the impressed voltage is 0, the initial charge on the capacitor is 10^{-5} coulomb, and the initial current is 0.

Find $Q(t)$, the charge on the capacitor, as a function of time.

(b) Now assume an impressed voltage (varying with time) given by

$$E(t) = 20\cos\omega t.$$

For what value of ω will the circuit's steady-state response be the greatest? What is the maximum voltage drop across the capacitor when this value of ω is used? Give exact values and decimal approximations.

C1. Use variation of parameters to find the general solution to the differential equation

$$y'' + 4y = \sec 2t.$$

C2. (B & D, §3.7, problem 17) Verify that $y_1 = x^2$ and $y_2 = x^2 \ln x$ (with $x > 0$) are solutions to the differential equation

$$x^2y'' - 3xy' + 4y = 0.$$

Find a particular solution the differential equation $x^2y'' - 3xy' + 4y = x^2 \ln x$.