A1. Let \( g(t) = \sum_{n=0}^{\infty} (-1)^n \delta(t - \pi n) \).

(a) Solve the IVP \( y'' + y = g(t) \); \( y(0) = 0, \ y'(0) = 0 \).
(b) Use a computer to plot your solution for at least \( 0 \leq t \leq 6\pi \).
(c) What is being modelled here? Predict the long-term behavior of the system.

A2. Repeat problem A1 with \( g(t) = \sum_{n=0}^{\infty} \delta(t - n\pi) \).

B1. (a) Solve the initial value problem
\[
y'' + 4y' + 13y = -5\delta(t - 1) + 3\delta(t - 2); \quad y(0) = 0, \quad y'(0) = 1.
\]
(b) Use the computer to plot your solution.
(c) Let \( y \) denote the solution to the IVP in B1a. From the picture in part B1b, you might guess that \( y \) is not differentiable at \( t = 1 \) or \( t = 2 \). However, \( y \) is continuous and (twice) differentiable everywhere else, so we can calculate one-sided derivatives at \( t = 1 \) and \( t = 2 \).
Find \( \lim_{t \to 1^{-}} y'(t) \), \( \lim_{t \to 1^{+}} y'(t) \), \( \lim_{t \to 2^{-}} y'(t) \), and \( \lim_{t \to 2^{+}} y'(t) \).
(d) Compute \( \lim_{t \to 1^{+}} y'(t) - \lim_{t \to 1^{-}} y'(t) \) and \( \lim_{t \to 2^{+}} y'(t) - \lim_{t \to 2^{-}} y'(t) \).
How are these numbers related to coefficients in the original differential equation?
C1. (B & D, §5.2, problem 3) Consider the differential equation $y'' - xy' - y = 0$. Suppose $y(x) = \sum_{n=0}^{\infty} a_n (x - 1)^n$ is a solution.

(a) Find the recurrence relation for the coefficients $a_n$.

(b) Find the first four (non-zero) terms in each of two linearly independent solutions.

C2. (B & D, §5.2, problems 7 and 17) Consider the differential equation $y'' + xy' + 2y = 0$. Suppose $y(x) = \sum_{n=0}^{\infty} a_n x^n$ is a solution.

(a) Find the recurrence relation for the coefficients $a_n$.

(b) Now impose the initial conditions $y(0) = 4$ and $y'(0) = -1$. Write the first five terms of the (power-series) solution to the resulting initial value problem.

(c) Plot the four-term and five-term approximations to the solution on the same axes. Estimate the interval on which the four-term approximation is reasonably accurate.