Math 333 Problem Set #9 Due 6 May 2003 (Optional), 12:00 NOON

Reading: B & D, §§6.3, 5.1, 5.2

Exercises: Write your solutions clearly, and in complete sentences, remembering that they will be graded for presentation as well as correctness.

A1. Let $g(t) = \sum_{n=0}^{\infty} (-1)^n \delta(t - \pi n)$.

(a) Solve the IVP $y'' + y = g(t); \quad y(0) = 0, \quad y'(0) = 0$.

(b) Use a computer to plot your solution for at least $0 \le t \le 6\pi$.

(c) What is being modelled here? Predict the long-term behavior of the system.

A2. Repeat problem A1 with $g(t) = \sum_{n=0}^{\infty} \delta(t - n\pi)$.

B1. (a) Solve the initial value problem

$$y'' + 4y' + 13y = -5\delta(t-1) + 3\delta(t-2); \quad y(0) = 0, \quad y'(0) = 1.$$

(b) Use the computer to plot your solution.

(c) Let y denote the solution to the IVP in B1a. From the picture in part B1b, you might guess that y is not differentiable at $t = 1$ or $t = 2$. However, y is continuous and (twice) differentiable everywhere else, so we can calculate one-sided derivatives at $t = 1$ and $t = 2$.

Find $\lim_{t \to 1^-} y'(t)$, $\lim_{t \to 1^+} y'(t)$, $\lim_{t \to 2^-} y'(t)$, and $\lim_{t \to 2^+} y'(t)$.

(d) Compute $\lim_{t \to 1^+} y'(t) - \lim_{t \to 1^-} y'(t)$ and $\lim_{t \to 2^+} y'(t) - \lim_{t \to 2^-} y'(t)$.

How are these numbers related to coefficients in the original differential equation?

C1. (B & D, §5.2, problem 3) Consider the differential equation $y'' - xy' - y = 0$. Suppose $y(x) = \sum_{n=0}^{\infty} a_n (x-1)^n$ is a solution.

(a) Find the recurrence relation for the coefficients a_n.

(b) Find the first four (non-zero) terms in each of two linearly independent solutions.

C2. (B & D, §5.2, problems 7 and 17) Consider the differential equation $y'' + xy' + 2y = 0$.

Suppose $y(x) = \sum_{n=0}^{\infty} a_n x^n$ is a solution.

(a) Find the recurrence relation for the coefficients a_n.

(b) Now impose the initial conditions $y(0) = 4$ and $y'(0) = -1$. Write the first five terms of the (power-series) solution to the resulting initial value problem

(c) Plot the four-term and five-term approximations to the solution on the same axes. Estimate the interval on which the four-term approximation is reasonably accurate.