Math 333 Solution
Quiz 1 6 February 2002

1. Find values for A, B, and C so that $y = At^2 + Bt + C$ is a solution to the differential equation $y'' - 2y' + 3y = 3t^2$.

   Solution: We have

$$\begin{aligned} y &= At^2 + Bt + C \\ y' &= 2At + B \\ y'' &= 2A. \end{aligned}$$

   Thus

$$\begin{aligned} y'' - 2y' + 3y &= 2A - 4At - 2B + 3At^2 + 3Bt + 3C \\ &= At^2 + (-4A + 3B)t + (2A - 2B + 3C). \end{aligned}$$

   Since $At^2 = 3t^2$, we know that $A = 1$. Next we know that $-4A+3B = 0$, so $3B-4 = 0$, and we have $B = \dfrac{4}{3}$. Finally, we have $2A - 2B + 3C = 0$, so that

$$2 - \frac{8}{3} + 3C = 0,$$

   and thus $C = \dfrac{2}{9}$.

2. Solve the initial value problem $y' = 3 - 2y$, $y(0) = 4$.

   Solution: We write

$$\frac{dy}{3 - 2y} = dt$$

   and integrate to get

$$\begin{aligned} -\frac{1}{2}\ln|3 - 2y| &= t + C_1 \\ \ln|3 - 2y| &= -2t + C_2 \\ 3 - 2y &= C_3 e^{-2t} \\ -2y &= C_3 e^{-2t} - 3 \\ y &= \frac{3}{2} + C_4 e^{-2t}. \end{aligned}$$

   From the initial condition, we find that $C_4 = \dfrac{5}{2}$, so the solution to the IVP is

$$y = \frac{3}{2} + \frac{5}{2}e^{-2t}.$$