Math 333 Solution
Quiz 2 13 February 2002

1. The air in a room with a volume of 100 cubic meters initially contains 5 grams of water vapor per cubic meter. At $t = 0$, a ventilation system starts pumping air into the room at the rate of 7 cubic meters per minute. The air from the ventilation system contains 15 grams of water vapor per cubic meter. It mixes with the air in the room, and the mixture is allowed to leave the room at the rate of 7 cubic meters per minute.

   Write (but do not solve) an initial-value problem for $W(t)$, the amount of water vapor in the room at time t. Be sure to specify units. (Assume that the temperature and atmospheric pressure in the room remain constant.)

   Solution: Let $W(t)$ denote the amount of water vapor in the room (in grams) at time t (in minutes). The rate at which water vapor is being added to the room is

$$7\frac{\mathrm{m}^3}{\mathrm{min}} \times 15\frac{\mathrm{g}}{\mathrm{m}^3} \quad = \quad 105\frac{\mathrm{g}}{\mathrm{min}}.$$

   The rate at which water vapor is being removed is

$$7\frac{\mathrm{m}^3}{\mathrm{min}} \times \frac{W(t)}{100}\frac{\mathrm{g}}{\mathrm{m}^3} \quad = \quad \frac{7W(t)}{100}\frac{\mathrm{g}}{\mathrm{min}}.$$

   The initial amount of water vapor in the room is $100 \times 5 = 500$ grams. The initial value problem is

$$W'(t) = 105 - \frac{7}{100}W(t); \quad W(0) = 500.$$

2. Solve the initial-value problem $(t^2 + 3)y' + 2ty = 4t + 5; \quad y(0) = 2.$

   Solution: The integrating-factor step is already done – the left-hand side of the equation is the derivative of $(t^2 + 3)y$. We have

$$\frac{d}{dt}(t^2 + 3)y \quad = \quad 4t + 5.$$

   We integrate to get

$$(t^2 + 3)y \quad = \quad 2t^2 + 5t + C.$$

   Using the initial condition, we get

$$3(2) \quad = \quad C,$$

   so $C = 6$, and the solution is

$$y(t) \quad = \quad \frac{2t^2 + 5t + 6}{t^2 + 3}$$