Math 333 | Solution
Quiz 4 | 14 March 2003

1. Solve the initial value problem $2y'' - 5y' - 3y = 0; \quad y(0) = 2,\ y'(0) = -1$.

   Solution: The characteristic equation is $2r^2 - 5r - 3 = 0$. We can factor this as $(2r+1)(r-3) = 0$, so the roots of the characteristic equation are $r = -\frac{1}{2}$ and $r = 3$. The general solution to the differential equation is

$$\begin{aligned} y &= c_1 e^{-\frac{t}{2}} + c_2 e^{3t} \\ y' &= -\frac{1}{2} c_1 e^{-\frac{t}{2}} + 3c_2 e^{3t}. \end{aligned}$$

   The initial conditions say that

$$\begin{array}{rcrcr} c_1 & + & c_2 & = & 2 \\ -\frac{1}{2}c_1 & + & 3c_2 & = & -1 \end{array}$$

   We multiply the second equation by 2 and add the result to the first to get $7c_2 = 0$, from which we conclude that $c_2 = 0$. So $c_1 = 2$ and the solution is

$$\boxed{y = 2e^{-\frac{1}{2}t}.}$$

2. Solve the initial value problem $y'' + 4y' + 13y = 0; \quad y(0) = 1,\ y'(0) = 0$.

   Solution: The characteristic equation is $r^2 + 4r + 13 = 0$, which has the complex roots $-2 \pm 3i$. The general solution to the differential equation is thus

$$\begin{aligned} y &= e^{-2t}(c_1 \cos(3t) + c_2 \sin(3t)) \\ y' &= -2e^{-2t}(c_1 \cos(3t) + c_2 \sin(3t)) + e^{-2t}(-3c_1 \sin(3t) + 3c_2 \cos(3t)). \end{aligned}$$

   The first initial condition says that $c_1 = 1$. Using this value for c_1, the second initial condition says that $-2 + 3c_2 = 0$, from which we get $c_2 = \frac{2}{3}$. The solution is

$$\boxed{y = e^{-2t}\left(\cos(3t) + \frac{2}{3}\sin(3t)\right).}$$