Math 333 | Solution
Quiz 5 | 28 March 2003

1. For each differential equation, find the correct form of a particular solution Y. Do not evaluate the constants.

   (a) $y'' + y = te^{-t} + \cos(t)$.

   Solution: The complementary solution contains a $\cos(t)$ term and a $\sin(t)$ term. To get the given forcing function, we'll need $t\cos(t)$ and $t\sin(t)$ as well as e^{-t} and te^{-t}. The form we want is

   $$Y = (A + Bt)e^{-t} + Ct\cos(t) + Dt\sin(t).$$

   (b) $y'' + y' = 2 + t^2e^t$.

   Solution: The complementary solution has the form $c_1 + c_2e^{-t}$. In order to get a constant in the forcing function, we'll need a term At in the particular solution. The function e^t does not appear in the complementary solution, so we can get t^2e^t in the usual way. The particular solution we want is

   $$Y = At + (Bt^2 + Ct + D)e^t.$$

2. Find the general solution to the differential equation $y'' + 2y' - 3y = t$.

   Solution: The complementary solution is $y_c = c_1e^t + c_2e^{-3t}$. The particular solution has the form

   $$Y = At + B.$$

   Plugging this into the given differential equation yields

   $$2A - 3(At + B) = t.$$

   We equate coefficients to get $A = -\frac{1}{3}$ and $B = -\frac{2}{9}$. The general solution is

   $$y = c_1e^t + c_2e^{-3t} - \frac{t}{3} - \frac{2}{9}.$$