1. A force of 10 N stretches a spring 2 cm beyond its natural length. A mass of 3 kg is attached to the spring and subjected to a varying force given by \( F(t) = 4 \cos(\omega t) \) N. For what value of \( \omega \) will the system resonate?

Solution:
The spring constant \( k \) is equal to \( 10 \text{ N}/0.02 \text{ m} = 500 \text{ kg/s}^2 \). The natural frequency of the system is

\[
\sqrt{\frac{k}{m}} = \sqrt{\frac{500}{3}} \text{ s},
\]

so the system will resonate when \( \omega = \sqrt{500/3} \) radians per second.

2. Consider the differential equation \( y'' + y' = g(t) \), where \( g(t) \) is an arbitrary function. Suppose that

\[
y = u_1 e^{-t} + u_2
\]

is a solution to the given equation. Using the method of variation of parameters, find formulas for \( u_1 \) and \( u_2 \) in terms of \( g \).

Solution: We have

\[
y' = -u_1 e^{-t} + u_1' e^{-t} + u_2'.
\]

We set \( u_1' e^{-t} + u_2' = 0 \). Then

\[
y'' = u_1 e^{-t} - u_1' e^{-t}.
\]

Adding \( y' \) and \( y'' \), we get

\[
y'' + y' = u_1 e^{-t} - u_1' e^{-t} - u_1 e^{-t} = -u_1' e^{-t}.
\]

Since we know \( y'' + y' = g(t) \), we get \( u_1' = -e^t g(t) \). Now we also know that \( u_2' = -u_1' e^{-t} \).

Substituting the known value for \( u_1' \), we get \( u_2' = g(t) \). The solution is

\[
u_1 = -\int e^t g(t) \, dt \quad u_2 = \int g(t) \, dt.
\]