Math 333 Solution
Quiz 6 11 April 2003

1. A force of 10 N stretches a spring 2 cm beyond its natural length. A mass of 3 kg is attached to the spring and subjected to a varying force given by $F(t) = 4\cos(\omega t)$ N. For what value of ω will the system resonate?

   Solution:

   The spring constant k is equal to 10 N/0.02 m = 500 kg/s^2. The natural frequency of the system is

   $$\sqrt{\frac{k}{m}} = \sqrt{\frac{500}{3}}\frac{1}{\mathrm{s}},$$

   so the system will resonate when $\omega = \sqrt{500/3}$ radians per second.

2. Consider the differential equation $y'' + y' = g(t)$, where $g(t)$ is an arbitrary function. Suppose that

   $$y = u_1 e^{-t} + u_2$$

   is a solution to the given equation. Using the method of variation of parameters, find formulas for u_1 and u_2 in terms of g.

   Solution: We have

   $$y' = -u_1 e^{-t} + u_1' e^{-t} + u_2'.$$

   We set $u_1' e^{-t} + u_2' = 0$. Then

   $$y'' = u_1 e^{-t} - u_1' e^{-t}.$$

   Adding y' and y'', we get

   $$\begin{aligned} y'' + y' &= u_1 e^{-t} - u_1' e^{-t} - u_1 e^{-t} \\ &= -u_1' e^{-t}. \end{aligned}$$

   Since we know $y''+y' = g(t)$, we get $u_1' = -e^t g(t)$. Now we also know that $u_2' = -u_1' e^{-t}$. Subsituting the known value for u_1', we get $u_2' = g(t)$. The solution is

   $$u_1 = -\int e^t g(t)\,dt \qquad u_2 = \int g(t)\,dt.$$