Math 333 | Solution
Quiz 7 | 18 April 2003

1. Consider the initial value problem $y''+4y'+11y = 3\sin 2t+1; \quad y(0) = 2, \quad y'(0) = -1.$ Find $Y(s)$, the Laplace transform of the solution to the IVP. Do not try to find $y(t)$ itself. Don't even simplify your expression for $Y(s)$ very much.

   Solution: Write $Y(s)$ for $\mathcal{L}\{y(t)\}$ The Laplace transform of the left-hand side is

$$\begin{aligned}\mathcal{L}\{y'' + 4y' + 11y\} &= s^2Y'' - sy(0) - y'(0) + 4(sY - y(0)) + 11Y \\ &= s^2Y'' - 2s + 1 + 4sY - 8 + 11Y \\ &= (s^2 + 4s + 11)Y - 2s - 7.\end{aligned}$$

   Meanwhile, on the right, we have

$$\mathcal{L}\{\sin 2t + 1\} = \frac{2}{s^2+4} + \frac{1}{s}.$$

   So here's the equation for $Y$. It's

$$\begin{aligned}(s^2 + 4s + 11)Y &= 2s + 7 + \frac{2}{s^2+4} + \frac{1}{s} \\ Y &= \frac{1}{s^2+4s+11}\left(2s + 7 + \frac{2}{s^2+4} + \frac{1}{s}\right).\end{aligned}$$

2. Find $\mathcal{L}^{-1}\left\{\frac{2s+1}{s^2+4s+13}\right\}$.

   Solution: We write

$$\begin{aligned}\frac{2s+1}{(s+2)^2+9} &= \frac{2(s+2)-3}{(s+2)^2+9} \\ &= \frac{2(s+2)}{(s+2)^2+3^2} - \frac{3}{(s+2)^2+3^2}.\end{aligned}$$

   The inverse Laplace transform is

$$2e^{-2t}\cos 3t - e^{-2t}\sin 3t.$$