Math 101 **Function Notation Worksheet** **September 2003**

Definition: A *function* is a rule f that assigns to each number x in a set D (called the *domain*) exactly one number $f(x)$.

In slightly more practical terms, this means that a function f takes an input number (or "argument"), x, and returns an output number, $f(x)$, that depends on x in an entirely predictable way.

Most of the functions we encounter in a calculus course are specified by algebraic formulas. To work with such a function, we give it a name (usually f, g, or h), and then write a *function definition* to associate that name with a particular formula.

Here's an example of a complete function definition:

Let f *be the function on the real numbers given by* $f(x) = x\sqrt{x^2+1}$.

In practice, we often leave out the specification of the domain (the phrase "on the real numbers" in the example above). We may even dispense with the "given by" construction and write simply (but not *quite* correctly)

Let $f(x) = x\sqrt{x^2+1}$.

The $f(x)$ on the left side of the function definition tells us that, throughout the definition, the input number will be denoted by x.

To determine what this function's output is when the input number has a specific value, say, when the input is 6, we substitute that value for every occurence of the variable x on the right side of the definition and simplify the result. The output we'd get from this particular function when the input value is 6 would be $6\sqrt{6^2+1}$ or $6\sqrt{37}$. Finding this value is called "evaluating f at 6," and it answers the following questions (among others):

- What is $f(x)$ when $x = 6$?
- What is $f(6)$?

Exercises I: Let f be the function given by $f(x) = \dfrac{3x-2}{x^2}$ and g be the function given by $g(x) = x(1-2x)$.

a) Find the value of $f(x)$ when $x = -1$.

b) Find $f(3)$.

c) Evaluate g at $x = 1$.

d) Find $g(1) + g(4)$.

e) Find $f(g(1))$.

f) Find $g(f(x))$.

The input to a function need not be an actual number; we can supply as input to a function any algebraic expression that represents a number. To find the output in this case, we proceed just as above. We substitute the input expression for each occurrence of the independent variable and simplify.

Example: Let f be the function given by $f(x) = \dfrac{x^2}{1-x}$. Find the value of $f(x)$ when $x = a + 2b$.
Solution: We have

$$\begin{aligned} f(a+2b) &= \frac{(a+2b)^2}{1-(a+2b)} \\ &= \frac{a^2+4ab+4b^2}{1-a-2b}. \end{aligned}$$

Note that we enclosed the expression $a + 2b$ in parentheses as we made the substitutions. This is important – always do it.

Exercises II:

Let $f(x) = x^2 + 2x$ and $g(x) = \dfrac{x}{(1+x)^2}$.

a) Find the value of $f(x)$ when $x = 2 + \delta$.

b) Find $g(3k)$.

c) Find $f(a + h) - f(a)$.

d) Find $g(x) + g(x + 1)$.

e) Find $f(x + 3)$.

f) Find $f(x) + 3$.