

1. Let $g(x) = \int_2^{x^3} \frac{1}{1 + \sqrt{t}} dt$. Find $g'(x)$.

Solution: By the Fundamental Theorem of Calculus Part I, we have

$$\begin{aligned} g'(x) &= \frac{1}{1 + \sqrt{x^3}} \cdot 3x^2 \\ &= \frac{3x^2}{1 + x^{\frac{3}{2}}} \end{aligned}$$

2. Compute $\int_1^3 \frac{t+4}{t^3} dt$.

Solution: We have

$$\begin{aligned} \int_1^3 \frac{t+4}{t^3} dt &= \int_1^3 t^{-2} + 4t^{-3} dt \\ &= \left[\frac{t^{-1}}{-1} + \frac{4t^{-2}}{-2} \right]_1^3 \\ &= \left[-\frac{1}{t} - \frac{2}{t^2} \right]_1^3 \\ &= \left(-\frac{1}{3} - \frac{2}{9} \right) - (-1 - 2) \\ &= \frac{22}{9} \end{aligned}$$

3. Find $\int x^2 e^{x^3+5} dx$

Solution: Let $u = x^3 + 5$. Then $du = 3x^2 dx$ so $\frac{1}{3} du = x^2 dx$ and the integral becomes

$$\begin{aligned} \frac{1}{3} \int e^u du &= \frac{1}{3} e^u + C \\ &= \frac{1}{3} e^{x^3+5} + C \end{aligned}$$