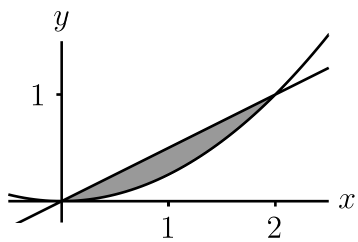
Math 202(01) Solution
Quiz 2 11 February 2004

1. Find the area of the region enclosed by the curves $y = \frac{x^2}{4}$ and $y = \frac{x}{2}$.

Solution: At right is a sketch of the region. To find the intersection points of the curves, we solve the equation

$$\frac{x^2}{4} = \frac{x}{2}$$



We cross-multiply to get $4x = 2x^2$, so we have

$$\begin{aligned} 0 &= 4x - 2x^2 \\ &= 2x(2-x) \end{aligned}$$

and the intersection points occur at $x = 0$ and $x = 2$.

The top curve is $y = x/2$ and the bottom curve is $y = x^2/4$, so the area is given by

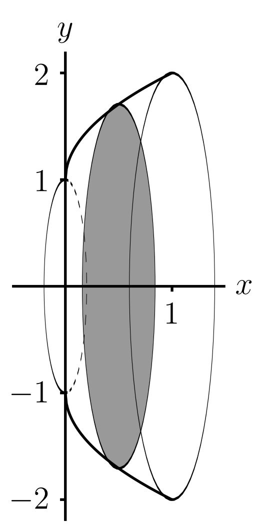
$$\begin{aligned} \int_0^2 \frac{x}{2} - \frac{x^2}{4}\,dx &= \left.\frac{x^2}{4} - \frac{x^3}{12}\right]_0^2 \\ &= \frac{4}{4} - \frac{8}{12} - (0-0) \\ &= \frac{1}{3} \end{aligned}$$

2. Find the volume of the solid obtained by rotating the region bounded by the curves $y = 1 + \sqrt{x}$, $x = 0$, $x = 1$ and $y = 0$ about the x-axis.

Solution: We'll use the disk method. At right is a sketch of the solid and a typical disk inside it.

The disk at position x_i^* has radius $1 + \sqrt{x_i^*}$, so its volume is

$$\pi(1 + \sqrt{x_i^*})^2\,\Delta x$$



Thus the volume of the whole region is given by

$$\begin{aligned} \int_0^1 \pi(1+\sqrt{x})^2\,dx &= \pi \int_0^1 1 + 2\sqrt{x} + x\,dx \\ &= \pi\left[x + \frac{4x^{\frac{3}{2}}}{3} + \frac{x^2}{2}\right]_0^1 \\ &= \pi\left[1 + \frac{4}{3} + \frac{1}{2} - (0)\right] \\ &= \frac{17\pi}{6} \end{aligned}$$