Math 202(01) Solution
Quiz 3 25 February 2004

1. A force of 5 N stretches a spring 4 cm beyond its natural length. How much work is done stretching the same spring from its natural length to 10 cm beyond its natural length?

   Solution: The spring constant is 5/4 Newtons per centimeter, or 500/4 Newtons per meter. The work done is either

$$\begin{aligned}\int_0^{10} \frac{5}{4} x\,dx &= \frac{5x^2}{8}\Big]_0^{10} \\ &= \frac{500}{8} = \frac{125}{2} \text{ N-cm}\end{aligned}$$

   or

$$\begin{aligned}\int_0^{\frac{1}{10}} \frac{500}{4} x\,dx &= \frac{500x^2}{8}\Big]_0^{\frac{1}{10}} \\ &= \frac{5}{8} \text{ N-m}\end{aligned}$$

2. Find $\int x^3 \ln x\,dx$.

   Solution: We use integration by parts. Let $u = \ln x$ so that $dv = x^3\,dx$. Then $du = \dfrac{dx}{x}$ and $v = \dfrac{x^4}{4}$. We have

$$\begin{aligned}\int x^3 \ln x\,dx &= \frac{x^4 \ln x}{4} - \int \frac{x^4}{4}\frac{dx}{x} \\ &= \frac{x^4 \ln x}{4} - \frac{x^4}{16} + C\end{aligned}$$

3. Evaluate $\int_0^{\frac{\pi}{2}} \sin^4 x \cos^3 x\,dx$.

   Solution: Write

$$\int_0^{\pi} \sin^4 x \cos^2 x \cos x\,dx = \int_0^{\pi} \sin^4 x(1 - \sin^2 x)\cos x\,dx$$

   New let $u = \sin x$, so that $du = \cos x\,dx$. When $x = 0$, we get $u = 0$, and when $x = \dfrac{\pi}{2}$, we get $u = 1$. The integral becomes

$$\begin{aligned}\int_0^1 u^4(1 - u^2)\,du &= \int_0^1 u^4 - u^6\,du \\ &= \frac{u^5}{5} - \frac{u^7}{7}\Big]_0^1 \\ &= \frac{1}{5} - \frac{1}{7} \\ &= \frac{2}{35}\end{aligned}$$