Math 202(01) Solution
Quiz 4 3 March 2004

1. Compute $\displaystyle\int_0^2 \frac{x-3}{(x+3)(x+1)}\,dx$. Give your answer in exact form.

   Solution: We write $\dfrac{x-3}{(x+3)(x+1)} = \dfrac{A}{x+3} + \dfrac{B}{x+1}$ and multiply through by the common denominator to get

$$\begin{aligned} x-3 &= A(x+1)+B(x+3) \\ &= (A+B)x + A + 3B \end{aligned}$$

   From this we get the system

$$\begin{array}{rcrcr} A & + & B & = & 1 \\ A & + & 3B & = & -3 \end{array}$$

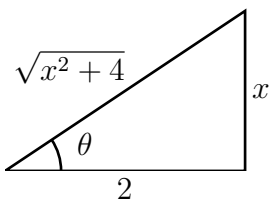
   Subtracting the second equation from the first gives $-2B = 4$, so the solution is $B = -2$ and $A = 3$. We have

$$\begin{aligned} \int_0^2 \frac{x-3}{(x+1)(x+3)}\,dx &= \int_0^2 \frac{3}{x+3} - \frac{2}{x+1}\,dx \\ &= 3\ln|x+3| - 2\ln|x+1|\big]_0^2 \\ &= (3\ln 5 - 2\ln 3) - (3\ln 3 - 2\ln 1) \\ &= 3\ln 5 - 5\ln 3. \end{aligned}$$

2. Find $\displaystyle\int \frac{dx}{(x^2+4)^{\frac{3}{2}}}$.

   Solution: We use the substitution $x = 2\tan\theta$, $dx = 2\sec^2\theta\,d\theta$ to get

$$\begin{aligned} \int \frac{2\sec^2\theta}{(4\tan^2\theta+4)^{\frac{3}{2}}}\,d\theta &= \int \frac{2\sec^2\theta}{(4\sec^2\theta)^{\frac{3}{2}}}\,d\theta \\ &= \frac{1}{4}\int \frac{\sec^2\theta}{\sec^3\theta}\,d\theta \\ &= \frac{1}{4}\int \cos\theta\,d\theta \\ &= \frac{1}{4}\sin\theta + C \end{aligned}$$



   Since $\tan\theta = \dfrac{x}{2}$, we get $\sin\theta = \dfrac{x}{\sqrt{x^2+4}}$. The integral is

$$\frac{x}{4\sqrt{x^2+4}} + C.$$