Math 202(01) Solution
Quiz 5 10 March 2004

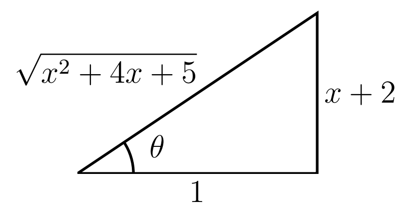
1. Find $\int \frac{x}{x^2+4x+5}\,dx$

   Solution: We complete the square in the denominator to get $\int \frac{x}{(x+2)^2+1}\,dx$, and use the substitution $x+2=\tan\theta$ (so that $dx=\sec^2\theta\,d\theta$) to rewrite the integral as

$$\begin{aligned}\int \frac{\tan\theta-2}{\tan^2\theta+1}\sec^2\theta\,d\theta &= \int \tan\theta - 2\,d\theta \\ &= \ln|\sec\theta| - 2\theta + C\end{aligned}$$

   To undo the substitution, we use the fact that $\tan\theta = \frac{x+2}{1}$ to get $\sec\theta = \sqrt{x^2+4x+5}$. The final result is

$$\ln\sqrt{x^2+4x+5} - 2\tan^{-1}(x+2) + C$$



2. Use the trapezoid rule with $n=20$ to estimate $\int_0^2 \sqrt{1+x^3}\,dx$. Give your answer to six decimal places.

   Solution: The subintervals begin and end at multiples of $\frac{1}{10}$, and the width of each subinterval is also $\frac{1}{10}$. Letting f denote the given function, we have

$$T_{20} = \frac{1}{10}\cdot\frac{1}{2}\left[f(0)+2f(1/10)+2f(2/10)+\cdots+2f(19/10)+f(2)\right]$$

   On the TI-85, we set `y1` to `(1+x^3)^(1/2)`, and then calculate

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sum seq(y1,x,1/10,19/10,1/10)
```

   which comes to about 30.429763928. We store this into the variable `A`. We note that $f(0)+f(2)=1+3=4$, so our next and final step is to compute

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(1/20)(4+2A)
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   The result is about 3.242976