Math 202(01) Solution
Quiz 6 31 March 2004

1. Rewrite $\int_e^\infty \frac{dx}{x(\ln x)^2}$ as a limit, and then evaluate it.

   Solution: We have

$$\begin{aligned}\int_e^\infty \frac{dx}{x(\ln x)^2} &= \lim_{t\to\infty}\int_e^t \frac{dx}{x(\ln x)^2}\\ &= \lim_{t\to\infty}\left[-\frac{1}{\ln x}\right]_e^t\\ &= \lim_{t\to\infty}\left(-\frac{1}{\ln t}+\frac{1}{\ln e}\right)\\ &= 1\end{aligned}$$

2. Evaluate $\int_0^3 \frac{1}{x^2-1}\,dx$

   Solution: Since the denominator is 0 at $x = 1$, this is an improper integral, and we need to write it as

$$\lim_{t\to 1^-}\int_0^t \frac{1}{x^2-1}\,dx + \lim_{t\to 1^+}\int_t^3 \frac{1}{x^2-1}\,dx$$

   To evaluate either of these, we need to find the indefinite integral

$$\begin{aligned}\int \frac{1}{x^2-1}\,dx &= \frac{1}{2}\int \frac{1}{x-1} - \frac{1}{x+1}\,dx\\ &= \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + C.\end{aligned}$$

   The first integral above is thus equal to

$$\lim_{t\to 1^-}\left[\frac{1}{2}\ln\left|\frac{x-1}{x+1}\right|\right]_0^t = \lim_{t\to 1^-}\ln\left|\frac{t-1}{t+1}\right| - \ln 1$$

   As $t \to 1^-$, we find that $\left|\frac{t-1}{t+1}\right| \to 0^+$, so the integral diverges.