Math 202(01) Solution
Quiz 8 14 April 2004

1. Assuming that the pattern of the first few terms continues, find the formula for the general term a_n of the sequence

$$\left\{\frac{1}{2}, -1, \frac{9}{8}, -1, \frac{25}{32}, -\frac{36}{64}, \frac{49}{128}, \ldots\right\}$$

Solution: The sequence is given by

$$a_n = (-1)^{n+1}\frac{n^2}{2^n}$$

2. Find $\lim_{n\to\infty} a_n$ if $a_n = 2 \cdot \dfrac{3^{n+1}}{5^n}$

Solution: We have

$$\begin{aligned}\lim_{n\to\infty} a_n &= 6 \lim_{n\to\infty} \left(\frac{3}{5}\right)^n \\ &= 6 \cdot 0 \\ &= 0\end{aligned}$$

3. Find the sum of the series $\dfrac{3}{5} - \dfrac{3}{25} + \dfrac{3}{125} - \dfrac{3}{625} + \cdots$

Solution: We rewrite the given series as

$$\begin{aligned}\frac{3}{5}\left(1 - \frac{1}{5} + \frac{1}{25} - \frac{1}{125} + \cdots\right) &= \frac{3}{5}\sum_{n=0}^{\infty}\left(\frac{1}{5}\right)^n \\ &= \frac{3}{5}\frac{1}{1-(-\frac{1}{5})} \\ &= \frac{1}{2}\end{aligned}$$