Math 202(01) — Solution

Quiz 9 — 28 April 2004

1. Determine whether each series is conditionally convergent, absolutely convergent, or divergent. Give reasons.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^n}{n4^n}$

   Solution: We test for absolute convergence using the ratio test. We have

   $$\begin{aligned} \lim_{n\to\infty} \frac{3^{n+1}}{(n+1)4^{n+1}} \frac{n4^n}{3^n} &= \lim_{n\to\infty} \frac{3n}{4(n+1)} \\ &= \frac{3}{4} \end{aligned}$$

   which is less than 1. So the series converges absolutely.

   (b) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$

   Solution: Since $\ln n$ is an increasing function of n, we know that $1/\ln n$ is decreasing. Also, we know that $\displaystyle\lim_{n\to\infty} \frac{1}{\ln n} = 0$. Thus by the alternating series test, the given series is convergent.

   Is it absolutely convergent? We have

   $$\frac{1}{\ln n} > \frac{1}{n}$$

   for all $n \geq 2$, and we know that $\sum \frac{1}{n}$ is divergent, so by the basic comparison test, the series is *not* absolutely convergent. Thus the given series is conditionally convergent.

2. Determine the interval of convergence for the power series $f(x) = \displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n (x+3)^n}{(n+1)2^n}$.

   Solution: We begin by applying the ratio test. We get

   $$\begin{aligned} \lim_{n\to\infty} \frac{|x+3|^{n+1}}{(n+2)2^{n+1}} \frac{(n+1)2^n}{|x+3|^n} &= \lim_{n\to\infty} \frac{|x+3|}{2} \frac{n+1}{n+2} \\ &= \frac{|x+3|}{2} \end{aligned}$$

   Setting $\frac{|x+3|}{2} < 1$, we get $|x+3| < 2$, so $-5 < x < -1$.

   Now we check the endpoints. At $x = -5$, we get $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n(-2)^n}{(n+1)2^n} = \sum_{n=0}^{\infty} \frac{2^n}{(n+1)2^n}$, which diverges, because it is the harmonic series.

   At $x = -1$, we get $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 2^n}{(n+1)2^n} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}$, the alternating harmonic series, which is convergent.

   The interval of convergence for the given series is $(-5, -1]$.