1. Let $T : \mathbb{P}_2 \rightarrow \mathbb{R}^2$ be a linear transformation from $\mathbb{P}_2$ the vector space of polynomials of degree 2 or less to the plane. Suppose that: $T(1) = (2, 1)$, $T(x) = (1, 4)$, and $T(x^2) = (5, 0)$.

   (a) If you encrypt the polynomials $a + bx + cx^2$ as vectors $(a, b, c)$, find the matrix formula that represents the linear transformation $T$.

   (b) Find a basis for the Kernel of $T$ and interpret it as a set of polynomials.

   (c) Find a basis for the image of $T$ in $\mathbb{R}^2$.

   (d) Is $T$ one-to-one? Is $T$ onto? Make sure to explain your reasoning.

   (e) What is the rank of $T$? What is the dimension of the null space of $T$. Explain your answers and give definitions for these two concepts.

2. Let $T : \mathcal{V} \rightarrow \mathcal{V}$ be a linear transformation from the subspace $\mathcal{V}$, a subspace of the vector space of all continuous functions from the real numbers to the real numbers, where $\mathcal{V} = \text{span}(e^x, \sin(x), \cos(x))$ to itself. Suppose that: $T(f(x)) = f'(x)$.

   (a) If you encrypt the polynomials $ae^x + b\sin(x) + c\cos(x)$ as vectors $(a, b, c)$, find the matrix formula that represents the linear transformation $T$.

   (b) Find a basis for the Kernel of $T$ and interpret it as the span of a set of functions in $\mathcal{V}$.

   (c) Find a basis for the image of $T$ in $\mathcal{V}$. 
(d) Is $T$ one-to-one? Is $T$ onto? Make sure to explain your reasoning.

(e) What is the rank of $T$? What is the dimension of the null space of $T$. What is the row space of $T$? Explain your answers and give definitions for these two concepts. What kinds of sets in $bfV$ are mapped to the same point in the image?

EXTRA PRACTICE FOR FINAL (NOT due for HMK)
1. Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^4$ be defined by

   \[
   T(x, y, z) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 0 & 0 & 5 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}.
   \]

   a. Find a basis for the range of $T$?
   b. Is $T$ onto? Explain clearly why or why not.
   c. Find a basis for the kernel of $T$.
   d. Is $T$ one-to-one? Why or why not.

2. Consider the vectors $u_1 = (1, 1, 1)$, $u_2 = (-1, 1, 0)$, and $u_3 = (1, 2, 1)$.

   (a) Use the formula $v \cdot w = ||v|| ||w|| \cos(\theta)$ to find the angles between $u_1$ and $u_2$, and $u_2$ and $u_3$. Use maple and the command: `evalf(arccos(1/(4*sqrt(7))))`; if you want to compute an arccosine value. Change the numbers.

   (b) Find the projection of $u_3$ onto $W$ the subspace spanned by $u_1$ and $u_2$.

   (c) Use Gram-Schmidt to transform $\{u_1, u_2, u_3\}$ into an orthonormal basis for $\mathbb{R}^3$. 