MA333: Homework #10
Due Friday, April 8

Read Chapter 3 sections 7-9

1. If \( y(t) \) gives the position of an object at time \( t \), then \( y''(t) \) is the object’s acceleration. Remember that Newton’s law states that acceleration is a constant (ie 1/m) times the force acting on the object. The initial value problem

\[
y''(t) = \cos(t) - \alpha y' \quad y(0) = 0, \quad y'(0) = 0
\]

models an object of mass 1 moving under the influence of an outside force \( \cos(t) \) and a second force \(-\alpha y'\) that pushes against the object’s motion, with a magnitude proportional to the objects velocity (assume \( \alpha > 0 \)). The second force models the friction of a sliding object with \( \alpha \) the coefficient of friction.

(a) Solve the initial value problem. All constants will depend on \( \alpha \).

(b) Use MAPLE to plot your solutions to this DE when \( \alpha = 0.25, \alpha = 4, \) and \( \alpha = 8 \). With each solution plot also plot the function \( \cos(t) \) (the forcing function). The maple command to plot two functions together is:

```
plot([\cos(t),\sin(t)], t=-8..8);
```

(c) Comment on the relation between the forcing function and the solution for the various values of \( \alpha \). How do the amplitude, the frequency, and the phase shift of the solution seem to depend on \( \alpha \)?

2. B & D, Section 3.7, problem 14

3. B & D, Section 3.7, problem 17

4. Use the method of variation of parameters to find the general solution to

\[
Y'' + 4y = \sec(2t)
\]

5. (a) Solve the initial value problem

\[
y'' + 2y' + 10y = 3\cos(3t) \quad y(0) = 0, \quad y'(0) = 1
\]

Identify which terms in your solution belong to the transient solution and which belong to the steady-state solution.

(b) Compare the amplitude and phase angle of the steady state solution with those of the function \( 3\cos(3t) \).

6. B & D, Section 3.8, problem 7

7. B & D, Section 3.8, problem 12