MA333: Homework #1 – SOLUTIONS
Short Week of January 26: Due Friday, January 28
A getting-to-know-you-homework – not to be done with parvanimity

Read Chapter 1 sections 1-4

Problem 2
a. Draw the direction field by hand for the first order differential equation in each problem. Use as a grid the integer lattice points in the $ty$-plane with $0 \leq t \leq 4$ and $0 \leq y \leq 4$.

Most people did this just fine.

b. Sketch in a few (say two) solution curves on each direction field.

Several people separated the variables and solved the DE exactly rather than just sketching a few solutions in on the direction field. Make sure you realize the difference so that you do not work too hard on a test or hmk in the future.

c. Try to predict the long-term behavior of all the solutions to the differential equation.

As $t$ increases, $y(t)$ exhibits 3 different long-term behaviors: $y(t)$ can increase to $+\infty$, decrease to $-\infty$, or remain constant at $y = 3/2$.

d. Look at different values of $y(0)$ (ie different values of $y$ when $t = 0$. How do the solutions differ for different values of $y(0)$.

For $y(0) = 3/2$ there is an equilibrium (or constant solution) of $y(t) = 3/2$. If $y(0) > 3/2$ then the solution curve $y(t)$ for that initial value increases exponentially to $+\infty$ as $t$ increases. If $y(0) < 3/2$ then the solution curve $y(t)$ for that initial value decreases exponentially to $-\infty$ as $t$ increases.

e. Use maple to graph the direction field and check that your field is accurate. make sure all your answers tally with the Maple generated direction field. Use it to catch your mistakes.

This was fine.

Problem 4
a. Most people did this just fine.

b. Several people separated the variables and solved the DE exactly rather than just sketching a few solutions in on the direction field. Make sure you realize the difference so that you do not work too hard on a test or hmk in the future.

c. As $t$ increases $y(t)$ exhibits 3 different long-term behaviors: $y(t)$ can decay exponentially with a horizontal asymptote at $-1/2$, increase with a horizontal asymptote at $-1/2$, or remain constant at $y = -1/2$. In all these cases, however,
\[ y(t) \text{ approaches } -1/2 \]

**Problem 19**

**a.** Most people did this just fine.

**b and c.** Sketch in curves. One interesting solution is \( y(t) = t - 3 \). The other solutions all approach this solution asymptotically as \( t \) increases. As \( t \) increases \( y(t) \) exhibits 3 different long-term behaviors: \( y(t) \) can approach \( y = t - 3 \) from above, approach it from below, or be the solution \( y(t) = t - 3 \) itself. In all these cases, however, \( y(t) \) approaches \( y = t - 3 \).

**d.** When \( y(0) = -3 \), you get the solution curve \( y(t) = t - 3 \). If \( y(0) > -3 \) then the solution curve \( y(t) \) for that initial value approaches \( y = t - 3 \) asymptotically from above as \( t \) increases. If \( y(0) < -3 \) then the solution curve \( y(t) \) for that initial value approaches \( y = t - 3 \) asymptotically from below as \( t \) increases.

**2.** Verify that \( y = te^{-2t} + t - 1 \) is a solution to \( y'' + 4y' + 4y = 4t \).

Note that \( y' = e^{-2t} - 2te^{-2t} + 1 \) and \( y'' = -4e^{-2t} + 4te^{-2t} \). Thus, \( y'' + 4y' + 4y - 4t = -4e^{-2t} + 4te^{-2t} + 4(e^{-2t} - 2te^{-2t} + 1) + 4(te^{-2t} + t - 1) - 4t = 0 \)