PRACTICE WITH MATRICES

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Identify the point \((x, y)\) in the plane with the column vector \(v = \begin{pmatrix} x \\ y \end{pmatrix}\). Then any matrix 
\[
A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}
\]
can be thought of as a linear transformation of \(\mathbb{R}^2\), taking \(v\) to \(Av\).

1. Compute \(v^Tv\), where \(v^T\) denotes the transpose of \(v\).
2. Find all matrices \(A\) satisfying \((Av)^T(Av) = v^Tv\).
3. Find the determinant of these matrices, that is, find \(\det A\).
4. Compute \(vv^T\).
5. Compute the trace of this matrix, that is, find \(\text{tr}(vv^T)\).
6. Compute the determinant of this matrix, that is, find \(\det(vv^T)\).

Think about what these results mean in \(\mathbb{R}^2\). Describe in words the result of question 1. What linear transformations correspond to the matrices you found in question 2. What is the significance of the determinant of \(A\) in question 3? Can you explain why the trace and determinant come out the way they do in the last two questions?